9 *Student Talk in a Problem-Centered Classroom*

The Problem Centered Learning (PCL) approach to teaching mathematics project is directed by Piet Human, Hanlie Murray, and Alwyn Olivier at the University of Stellenbosch in South Africa. This is a large, multifaceted project including curriculum development and teacher training. A focus of the initial work was addition and subtraction of multidigit numbers, but it has expanded well beyond that now.

Computation is viewed as a vehicle that students can use to increase their understanding of number and the properties of numbers and operations. In problem-centered learning classrooms, students are presented with computation problems that are meaningful and interesting to them, but which they cannot solve with ease using routinized procedures or drilled responses. The teacher does not demonstrate a solution method, nor does the teacher indicate a preferred method, yet she or he expects every student to become involved with the problem and to attempt to solve it. Students' own invented methods are expected and encouraged. It is expected of students to discuss, critique, explain, and when necessary, justify their interpretations and solutions.

Students use methods for solving addition and subtraction problems that are based directly on their conceptions of number. Learning activities for number topics are aimed at helping students construct increasingly sophisticated meanings for number. For young children, a number like 37 means only 37 single objects. There is no understanding of the number in terms of tens and ones. Later, students realize they can make 37 by counting by tens (10, 20, 30) and then counting 7 ones. Still later, they see that 37 is made up of 3 tens and 7 ones. These are major conceptual developments and the goal of instruction is to help students experience these developments. Young students in PCL classrooms engage in a variety of counting activities. They count large sets of objects into groups, beads on a ten-frame, and numbers on a hundreds-chart. They become especially skilled at counting in tens. Consequently, many of their invented addition and subtraction methods are based on counting. The way in which students build their procedures from their conceptions of number is therefore somewhat different from students in some of the other projects described here, who develop addition and subtraction methods by building from their manipulations of physical materials.

Computation activities in the PCL classrooms involve presenting small groups of students with a word problem. Students are expected to work individually or as a group to develop a method to solve the problem and then to present their method to their peers and the teacher. The teacher provides suggestions for notation and terminology when needed, but does not demonstrate particular solution methods. By reflecting on the methods of others and working out ways of communicating their own methods, students are expected to develop increasingly advanced conceptions of number.

In the upper elementary grades (fourth grade and higher) topics such as common and decimal fractions are presented similarly, the main difference being that teachers find it more difficult to construct suitable tasks (learning environments) for these topics than for whole number computation.

In the classroom, different events may occur in different sequences. The teacher may elect to work with only part of her or his class, or the teacher may work with the whole class. These decisions are influenced by the ages of the students, the number of students in the class, and the mathematics that she or he wishes to address. Presenting students with a challenging problem which they are expected to solve without teacher assistance is the most frequently used learning episode.

When the students are slightly older (e.g., fourth grade and onward) a popular operation is for the teacher to pose a problem to the whole class, make sure that they understand the problem, and then ask them to continue. While students are solving the problem they are expected to interact with each other in whatever fashion they need at a particular point in the problem-solving process. For example, some students may initially work individually and only after having reached a tentative solution feel the need for discussion and sharing; at other times they may choose to discuss the problem first, reach consensus about a solution method, and then proceed as above.

During the problem-solving phase, interaction among students is informal and voluntary, its form dictated by the nature of the task (and especially its level of difficulty) and the individual student's needs. Students may be working in pairs or in clusters of four, assigned by the teacher or according to the students' choices. Since the groups are not cooperative learning groups in the sense that each group is required to reach consensus on an answer, many teachers do not make use of any

physical grouping structure, but simply encourage students to move around and talk to or collaborate with whom they wish. The main emphasis is on *personal* construction of meaning within a community which supports discourse and the construction of shared meanings: the process of constructing shared meanings should not be perceived as more desirable than (and occurring at the expense of) personal understanding (Murray, Olivier, and Human 1993). Social interaction is seen as conducive to personal construction in that it encourages reflection, the identification and correction of mistakes, and the development of concepts.

While students are solving the problem, the teacher moves among them, but limits her or his role to that of facilitator. When some students have solved the problem, the teacher may give those students a new problem. When everybody has solved the first problem, the teacher may initiate a general class discussion on the different methods used, or decisions taken and the nature and reasons for some mistakes students may have made. This general discussion has a more formal nature than the student interaction which takes place while the problem is being solved.

For some problems, students work on their own (independently of the teacher) for a considerable length of time. What happens when PCL students are working on their own? What is the quality of young students' mathematical thinking when they are working independently in peer groups?

To investigate this question we videotaped several groups of students in the classroom who were left solving mathematical problems on their own. All of these sessions showed a similar pattern of development and similar interaction patterns among the students themselves. Therefore, it seems fair to assume that these episodes are typical of those classrooms where similar social contracts have been established. Videotaped recordings and personal observations of other classrooms bear this out.

A Problem-Solving Session

The following session consisting of four students solving a problem without teacher intervention was recorded in a fourth-grade classroom during the eighth month of the school year. The school is situated in a large country town. The teacher, Ms. Lombard, has taught at the school for many years and has participated in inservice teacher education sessions for PCL during the past five years. These sessions took the form of a small number of short workshops organized by the local education authority, followed by a program of quarterly meetings initiated by the teachers in the area. During these meetings teachers share ideas about matters like resources and assessment. Ms. Lombard uses a textbook designed for problem-centered learning as her main source of tasks. She regards this class as of above average ability, and the students recorded in this session as average for the class.

The class period was fifty minutes long. The recorded session lasted forty minutes. Ms. Lombard presented the following problem verbally to the class: Mom makes small apple tarts, using three-quarters of an apple for each small tart. She has 20 apples. How many small apple tarts can she make?

Ms. Lombard selected this problem so that students could become familiar with its structure (a grouping or measurement type of division problem where the group size is a fraction). Many students and even adults cannot solve this problem when they are required to identify the operation and perform it (20÷3/4) to reach a solution. Yet this problem can be solved in a variety of ways through a variety of strategies when students are encouraged to analyze the physical situation and construct methods that fit their interpretation of the situation. A class discussion of these different conceptualizations of the problem and the methods that are based on them help students to gain a deeper understanding of the problem.

The students started working on the problem in small voluntary groups. Ms. Lombard sometimes withdrew completely to work at her desk, and at other times moved around in the classroom, asking questions and challenging students. One of the groups of students was videotaped. (Ms. Lombard was requested not to approach them.) This is a transcription of the videotape. The few instances in which the conversation on the tape was not clearly audible have been omitted.

During the first phase, lasting about fifteen minutes, there is comparatively little conversation and little interaction; the students would often think aloud. For example:

NINA: This is going to take too long—Ouch! SSh!

JEANETTE: This is going to take too long.

NINA: I mean this way is going to take too long. I think I'd better take another way. Have you got the answer?

(Laughter)

NINA: Twenty apples, so that's at least twenty milk tarts. . . . But now, she has—Oh, I got the answer!

JEANETTE: Will you please just keep quiet.

NINA: Almost, almost . . . I just have to add it up.

Each student then gives the answer that she has obtained, and Jeanette and Nina explain their methods. Although Nina has made an obvious and common computational error, the other three students are so uninvolved at this stage that they do not notice it, and an impasse is reached. The video recording shows feelings of helplessness and amusement at their predicament. The discussion preceding this point went as follows:

NINA: OK, Kerri, what's your answer?

KERRI: No, Liz, what's yours?

JEANETTE: I got that she could make fifteen apple tarts.

KERRI: So did I.

LIZ: Because what I did twenty divided by 4 to find out what one-third is . . . a fourth is, and then I timesed it by three and I got fifteen.

NINA: It's the wrong way, I think.

KERRI: What did you do?

NINA: See I—what I did she has twenty apples. She only needs three-quarters to make one apple tart, so she should at least make twenty apple tarts . . . OK.

(Laughter)

NINA: If you don't understand that, don't worry. Then there are twenty apples left, twenty quarters left over, and so I thought I would take *three* . . . plus three plus three plus three plus three plus three and I got eighteen, and I plussed the eighteen and the twenty and got 38 as my answer.

The error Nina makes here is to take the sum of the six threes as the answer to 20÷3, instead of the *number* of threes.

KERRI: So they could make 38 apple tarts.

NINA: That's what I got. Don't you agree?

JEANETTE: Twelve . . . three-quarters. Three-quarters plus three-quarters is equal to one and a half.

NINA: You can go three times four is twelve.

JEANETTE: Oh, I see what was my mistake

LIZ: Why did you say that they got twenty apple tarts?

NINA: Well you see twenty here, milk tarts, I wrote as opposed to apple tarts, she's got to use three-quarters of an apple tart, of an apple to make an apple tart, you understand that. She can at least make twenty.

KERRI: But how do you know she can at least make twenty?

NINA: Because she has twenty apples. You don't seem to understand me. And then twenty quarters left over so another three-fourths are going to make one apple tart, so plus three plus three plus three plus three plus three plus three is eighteen, plus eighteen plus twenty is 38.

KERRI: I don't quite follow your method.

NINA: Well . . . what do we do next?

LIZ: I don't know.

After some time, the deadlock is broken by Liz.

LIZ: Must I explain my method to you again?

Here follows some very polite but, at first glance, not very mathematically productive conversation:

LIZ: Well, what have *you* done? Because maybe yours is the right way.

NINA: I think your answers are wrong but mine might not be right.

JEANETTE: She's got a point there.

Immediately after this Nina suggests that they draw the apples, which they all start doing, although with some protest. This seems to indicate that the preceding few minutes' polite exchanges actually did prepare them to involve themselves with the problem.

A phase of intense activity starts. Only at this point does it seem as if the real thinking and argument take off.

NINA: Should we draw the apples?

LIZ: *You* can draw them.

NINA: One apple, one apple . . . I think we should just draw one apple.

KERRI: You don't have to do many.

NINA: Three-quarters of an apple.

LIZ: I'll do it as well.

JEANETTE: Guys, I've got twenty-three.

KERRI: Twenty-three? But how did you get that?

KERRI: I'm going to work it out again.

NINA: Mine is right! I've worked it out! My answer is right!

KERRI: But how do you know, because you've only drawn one apple!

NINA: But I've worked it out! Look, I'll show you—here, see, I've used that three and there's one left, twenty apples and one left. So there are twenty of that.

KERRI: I think you've got yours right, but I'm just going to work it out again to see. . . . How did you get—

LIZ: I think I've got the answer and our answer might be wrong.

KERRI: I'm listening.

JEANETTE: In each apple there is a quarter left. In each apple there is a quarter left, so you've used, you've made twenty tarts already and you've got a quarter of twenty see—

LIZ: So you've got twenty quarters *left*.

JEANETTE: Yes, . . . and twenty quarters is equal to five apples, . . . so five apples divided by—

LIZ: Six, seven, eight.

JEANETTE: By three-quarters equals three.

KERRI: But she can't make only three apple tarts!

JEANETTE: No, you've still got twenty.

LIZ: But you've got twenty quarters, if you've got twenty quarters you might be right.

JEANETTE: I'll show you.

LIZ: No, I've drawn them all here.

KERRI: How many quarters have you got? Twenty?

LIZ: Yes, one quarter makes five apples and out of five apples she can make five tarts which will make that twenty-five tarts and then she will have, wait, one, two, three, four, five quarters, she'll have one, two, three, four, five quarters. . . .

NINA: I've got a better . . .

KERRI: Yes?

LIZ: Twenty-six quarters and a remainder of one quarter left.

Nina remains outside this discussion, because she still believes that she has the correct solution. She was the only one who originally interpreted the structure of the problem correctly, but made a computational error. It may be that her knowledge that the others' original interpretations were definitely wrong now prevents her from being willing to listen to them.

Nina tries to rejoin the discussion, but the other three seem to have established consensus about the basic correctness of their approach and Liz and Jeanette are trying to construct a coherent explanation for Kerri's benefit, who is listening intently.

NINA: I've got another way, see my answer will be wrong because that is *eighteen*, minus four, so that is fourteen, that's one whole, minus four is ten minus four is six minus four is two, OK? Then I have a remainder of two apples but I have an answer of one whole, two wholes, three wholes, four wholes, I've got four wholes, it's twenty-four remainder two.

Nina is now trying to make whole apples from the remaining quarters, but takes 18 as the number of remaining quarters and not 20. The other three students are so involved in their own discussion that they are not prepared to listen to her at this point. Liz has formulated exactly the same strategy of dealing with the remaining quarters, but (correctly) using 20 quarters:

LIZ: No, can I show you my way?

NINA: Or 38.

LIZ: What I've done, I've done all those then you've got twenty quarters left, then you do twenty divided by one-fourth and you get five apples.

NINA: Twenty-four.

LIZ: But then I'll have five apples left if you put them all together.

KERRI: What? Explain again.

LIZ: Twenty divided by one-quarter would make it five quarters. So then, she'd make one tart and she'd have two quarters left over which would make it twenty-one. . . .

NINA: Don't change your answers now, just leave them how they are.

Nina is clearly not willing to listen.

LIZ: But I understand where I went wrong so she could only make twenty-one.

KERRI: Look, I've drawn twenty apples.

NINA: But she could make twenty-four.

KERRI: Why?

JEANETTE: Guys, can I explain?

KERRI: Yes?

JEANETTE: See, you've got twenty apples—You only use three-quarters of one apple so you obviously have to have more than twenty.

KERRI: Oh, yes. Look, I must color mine.

JEANETTE: You've got twenty quarters left and twenty divided by one-fourth is equal to five.

LIZ: Which would make it five apples.

JEANETTE: Yes, it's right, it's five *apples*.

LIZ: So she could make twenty-four tarts and she'd have a remainder of one quarter. Yes.

JEANETTE: *No* . . . a remainder of two quarters.

LIZ: Because look here, if you take the five apples . . .

Jeanette: You've got twenty quarters.

LIZ: Yes.

JEANETTE: So she can make twenty-five apple tarts.

NINA: I *still* think she can make twenty-four remainder two.

LIZ: You've done all the apples and you'd have twenty apple tarts and you'd have twenty quarters left over and now if you divide the twenty into quarters then you'd have five full apples, and five full apples you can make five tarts and then another one from the left-over quarters and then you'd have a remainder of two quarters.

JEANETTE: Just hold on. . . . I know what's wrong . . . one, two, three, four, five.

LIZ: Do you see?

KERRI: Not *exactly*.

JEANETTE: . . . And you've still got one, two, three, four, five quarters. So you've already made twenty-five, you've still got five quarters and here you only use three quarters so you can make *twenty-six* remainder two.

LIZ: Yes.

JEANETTE: And you have half left, and you can store it in your Deepfreeze or your refrigerator or you—

NINA: I still say twenty-four remainder two quarters, remainder two *quarters, then* you say *twenty-five* remainder two quarters, no, *now* you say *twenty-six* remainder two quarters, now what next?

This last (possibly sarcastic) remark of Nina's elicits a direct response from Liz and Jeanette, but Nina counters with another sarcastic remark, and Nina then turns to Kerri, who has committed herself to trying to understand their reasoning.

LIZ: Here you've got the apples and you cut them into quarters, into three quarters and you can make tarts and then you've got twenty quarters left over and then you do twenty quarters . . . .

JEANETTE: And then you've got twenty quarters, that's twenty divided by one-fourth is equal to five.

LIZ: So you've got five full apples, and then you can make . . . oopsie . . . and then you can make five apple tarts with those five and you can make another one with the quarters, with the remainder of . . .

NINA: But there's six quarters left.

LIZ: Do you see it's twenty-six remainder two?

NINA: Do you want to hear all the answers—fifteen, thirty-eight, twenty-two remainder two quarters, twenty-four remainder two quarters, twenty-five remainder two quarters and twenty-six remainder two quarters!

LIZ: But do you understand why you have twenty-six remainder two quarters?

KERRI: No, but I get twenty, twenty, one, two, three, four, five, then—

LIZ: Then you've got five quarters left over.

JEANETTE: Yes, but then you've got five quarters left so that you can make another one. So then you've got a remainder of a half an apple.

KERRI: Yeah.

Discussion

As described in the previous chapters, there are certain features of classrooms that determine the nature of a learning episode like the one described and that account for its success. We will highlight several features that follow along these dimensions: the nature of the task, the role of the teacher, and the social culture of the classroom.

The Nature of the Task

An episode like this is initiated and sustained by tasks that are genuine problems for students, tasks that offer opportunities for students to perceive problems that they need to solve. The task provides the opportunity, but the students must set the goal of completing the task for the task

to become problematic. In this episode, students were initially following the teacher's instructions to solve the problem. They had not really adopted this goal as their own. But shortly after the point where Nina suggests that they draw the apples, they show that they have taken ownership of the problem and have made a personal commitment to solve it. It is only at this point that they begin to seriously reflect on the mathematics of the task.

In many traditional mathematics classrooms, it is unusual for fourth graders to work together on solving a problem for forty minutes—or even ten minutes. Most problems can be solved quite quickly, in two or three minutes or less. What enables students to work for prolonged periods of time solving a single problem? An appropriate task is surely part of the reason. The task of apples and tarts, although not fancy or elaborate, connected well with students' current level of functioning. They understood the task and already had developed some methods with which they could begin, but the task was challenging and their methods needed to be rethought and reshaped. It is a mistake, though, to think that the task alone encourages students to work for forty minutes. Teachers who simply change tasks will not find students suddenly working independently for long periods of time. Other dimensions of this instructional system, such as the role of the teacher and the social culture of the classroom, are equally important.

The Role of the Teacher

After the teacher has presented a suitable task, she or he should allow the students to work on the task without continual interruptions and interference. Even teachers who wholeheartedly accept this as a principle do not always realize the extent to which students can be trusted to resolve their own dilemmas while they are struggling with a problem.

Many teachers may have felt compelled to intervene in this episode and resolve Nina's doubts by pressing for consensus, but constructing mathematical knowledge and reaching true understanding are deeply personal processes which are very sensitive to interferences. A lack of time and a quiet space in which to think may delay these processes. Since Nina was the first student to grasp the basic structure of the problem and actually set the others off on the right track, it seems amazing that she could not reach the correct answer. Her way of dealing with the remaining 20 quarters was initially quite different from theirs—she wanted to find the number of threes in twenty, whereas they put together 20 quarters to make whole apples. When Nina decided to use this approach as well, the other students were not aware of it. It is possible that if Nina had allowed herself some more time for reflection, she would have been able to correct her error.

This sensitivity to the process of constructing knowledge and understanding is illustrated even more powerfully by the way the impasses are dealt with. The students' progress came to an apparent standstill two or three times in the course of solving the problem, and although these periods lasted for several minutes, the students themselves broke the impasse each time. We have observed that many teachers have a low threshold of tolerance for this type of situation, and want to intervene with a suggestion to get a group going again. Many teachers feel it is their responsibility to clear up immediately any confusion and remove any obstacle to progress. This episode suggests that teachers should think carefully before they intervene—students may frequently be able to resolve impasses by themselves. By intervening too often or too quickly teachers may undermine the social culture they are trying to build.

The Social Culture of the Classroom

A feature that heavily influenced this episode is the social contract which underlies the interactions between students and teacher and, in this case, between student and student. The episode illustrates many of the points made in Chapter 4; we will review two of them here.

The first point to consider is the importance of mistakes: Mistakes are sites for learning. As Nina observed sarcastically, the whole episode is studded with incorrect answers. These wrong answers sometimes were simply the result of incomplete or ill-formed thinking, but sometimes they served as stepping stones as students built their solutions. One might even say that a learning sequence of the kind in the episode would be unlikely without the mistakes that occurred along the way. The mistakes often signaled differences in students' opinions and these differences generated the arguments and the attempts to justify and explain. And these justifications and explanations provided the real learning opportunities, both for the speaker and for the listeners.

The second feature to consider is how communication skills develop and enable further learning. This episode illustrates how students use mathematical language to communicate. Through much of the episode, students used mathematically incorrect language. Phrases like "twenty divided by a quarter give five" were common. This does not seem to have hampered their communication, possibly because they still had the physical situation in mind. Eventually, though, the students themselves experienced the need to express their thoughts as clearly and coherently as possible. In the end, the (correct) solution method is described four times, with increasing clarity and conviction.

Even when students are sure that they have reached the correct answer, and know how they have reached it, they need to articulate their thinking repeatedly. In this episode it can be argued that the explanations were for Kerri's benefit, but there is a strong impression that the explanations actually served to stabilize the speaker's own conceptions. It is common for students in problem-centered classrooms to engage in explanations and justifications to promote reflection and to establish a social culture of intellectual risk taking, a tolerance for mistakes, and of mutual respect. However, this episode illustrates how strongly students themselves need to construct a coherent, exact account of their thinking. As Lampert (in press) has noted, understanding something is at least partly a function of helping others understand it.

Conclusion

During this forty-minute episode, four students who are familiar with inquiry learning and have participated in the system of instruction we are describing try to solve a problem through personal engagement, argument, and reflection. They feel free to hold their own opinions, change their minds, build on others' thinking, invent their own methods, and adopt the methods of others. This is all a natural part of the problem-solving process.

Earlier we asked the question "What happens when students are working on their own?" The episode illustrates what can happen and our observations suggest that the episode is typical for students in the kinds of classrooms we have described. What this means is that students can reason mathematically and construct important understandings without teacher intervention. This should not be interpreted as a conclusion that teachers should never intervene, nor is it a conclusion that this is the only kind of learning situation that is productive. Rather, the conclusion is that students can take responsibility for their own learning provided the task is suitable and the social culture is of such a nature that students know this kind of behavior is expected. The message is that students can work well independently and teachers should not assume too quickly that intervention is the best choice.