

IMPLEMENTING STANDARDS-BASED MATHEMATICS INSTRUCTION

A Casebook
for Professional
Development

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Chapter 1

ANALYZING MATHEMATICS INSTRUCTIONAL TASKS

Mathematical tasks can be examined from a variety of perspectives including the number and kinds of representations evoked, the variety of ways in which they can be solved, and their requirements for student communication. In this book, we examine mathematical instructional tasks in terms of their cognitive demands. By cognitive demands we mean the kind and level of thinking required of students in order to successfully engage with and solve the task.

In this chapter, we describe a method for analyzing the cognitive demands of tasks as they appear in curricular or instructional materials (the first phase of the Mathematical Tasks Framework shown in Figure I.3 in the Introduction). Unlike the remainder of the framework, which describes task evolution *during* a classroom lesson, the initial phase of the framework focuses on tasks *before* the lesson begins, that is, the task as it appears in print form or as it is created by the teacher.

Why are the cognitive demands of tasks so important? As stated in the *Professional Standards for Teaching Mathematics* (NCTM, 1991), opportunities for student learning are not created simply by putting students into groups, by placing manipulatives in front of them, or by handing them a calculator. Rather, it is the level and kind of thinking in which students engage that determines what they will learn. Tasks that require students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking: tasks that demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas lead to a different set of opportunities for student thinking. Day-in and day-out, the cumulative effect of students' experiences with instructional tasks is students' implicit development of ideas about the nature of mathematics—about whether mathematics is something they personally can make sense of, and how long and how hard they should have to work to do so.

Since the tasks with which students become engaged in the classroom form the basis of their opportunities for learning mathematics, it is important to be clear about one's goals for student learning. Once learning goals for students have been clearly articulated, tasks can be selected or created to match these goals. Being aware of the cognitive demands of tasks is a central consideration in this matching. For example, if a teacher wants students to

learn how to justify or explain their solution processes, she should select a task that is deep and rich enough to afford such opportunities. If, on the other hand, speed and fluency are the primary learning objectives, other types of tasks will be needed. In this chapter, readers will learn how to differentiate among the various levels of cognitive demand of tasks, thereby laying a foundation for more careful matching between the tasks teachers select for the classroom and their goals for student learning.

DEFINING LEVELS OF COGNITIVE DEMAND OF MATHEMATICAL TASKS

The example shown in Figure 1.1 illustrates four ways in which students can be asked to think about the relationships among different representations of fractional quantities. Each of these ways places a different level of cognitive demand on students. As shown on the left side of the figure, tasks with lower-level demands would consist of memorizing the equivalent forms of specific fractional quantities (e.g., $1/2 = .5 = 50\%$) or performing conversions of fractions to percents or decimals using standard conversion algorithms in the absence of additional context or meaning (e.g., convert the fraction $3/8$ to a decimal by dividing the numerator by the denominator to get .375; change .375 to a percent by moving the decimal point two places to the right to get 37.5%). These lower-level tasks are classified as *memorization* and *procedures without connections to understanding, meaning, or concepts* (hereafter referred to simply as *procedures without connections*), respectively. When tasks such as these are used, students typically work 10–30 similar problems within one sitting.

Another way in which students can be asked to think about the relationships among fractions, decimals, and percents—one that presents higher-level cognitive demands—might also use procedures, but do so in a way that builds connections to underlying concepts and meaning. For example, as shown in Figure 1.1, students might be asked to use a 10×10 grid to illustrate how the fraction $3/5$ represents the same quantity as the decimal .6 or 60%. Students would also be asked to record their results on a chart containing the decimal, fraction, percent, and pictorial representations, thereby allowing them to make connections among the various representations and to attach meaning to their work by referring to the pictorial representation of the quantity every step of the way. This task is classified as *procedures with connections to understanding, meaning, or concepts* (hereafter referred to simply as *procedures with connections*).

Another high-level task (classified as *doing mathematics*¹) would entail asking students to explore the relationships among the various ways of representing fractional quantities. Students would not—at least initially—be provided with the conventional conversion procedures. They might once again use grids,

Lower-Level Demands

Memorization

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected Student Response:

$$\frac{1}{2} = .5 = 50\%$$

$$\frac{1}{4} = .25 = 25\%$$

Procedures without connections

Convert the fraction $\frac{3}{8}$ to a decimal and a percent.

Expected Student Response:

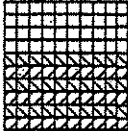
Fraction	Decimal	Percent
$\frac{3}{8}$.375	37.5%
	$8 \overline{) 3.000}$	
	24	
	60	
	56	
	40	
	40	

Higher-Level Demands

Procedures With Connections

Using a 10×10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.

Expected Student Response:

Pictorial	Fraction	Decimal	Percent
	$\frac{60}{100} = \frac{3}{5}$	$\frac{60}{100} = .60$	$.60 = 60\%$

Doing Mathematics

Shade 6 small squares in a 4×10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of area that is shaded, b) the decimal part of area that is shaded, and c) the fractional part of area that is shaded.

One Possible Student Response:

Fraction	Decimal	Percent
$\frac{6}{40}$.15	15%

a) One column will be 10% since there are 10 columns. Then 2 squares is half a column and half of 10% which is 5%. So the 6 shaded blocks equal 10% plus 5% or 15%.

b) One column will be .10 since there are 10 columns. The second column has only 2 squares shaded so that would be one half of .10 which is .05. So the 6 shaded blocks equal .1 plus .05 which equals .15.

c) Six shaded squares out of 40 squares is $\frac{6}{40}$ which reduces to $\frac{3}{20}$.

FIGURE 1.1. Lower-level vs. higher-level approaches to the task of determining the relationships among different representations of fractional quantities (Stein & Smith, 1998). (Reprinted with permission from *Mathematics Teaching in the Middle School*, copyright 1998 by the National Council of Teachers of Mathematics. All rights reserved.)

but this time grids of varying sizes (not just 10×10) would be used. As shown in Figure 1.1, students could be asked to shade six squares of a 4×10 rectangle and to represent the shaded area as a percent, a decimal, and a fraction. When students use the visual diagram to solve this problem, they are challenged to apply their understandings of the fraction, decimal, and percent concepts in novel ways. For example, once a student has shaded the six squares, he or she must determine how the six squares relate to the total number of squares in the rectangle. In Figure 1.1, we see an example of a student's response to this task that illustrates the kind of mathematical reasoning used to come up with an answer that makes sense and that can be justified. In contrast to the tasks with lower-level demands discussed earlier, in *procedures-with-connections* or *doing-mathematics* tasks, students typically perform far fewer problems (sometimes as few as two or three) in one sitting.

MATCHING TASKS WITH GOALS FOR STUDENT LEARNING

As illustrated by the above discussion, not all mathematical tasks provide the same opportunities for student learning. Some tasks have the potential to engage students in complex forms of thinking and reasoning while others focus on memorization or the use of rules or procedures. In our work with teachers in the QUASAR Project, we discovered the importance of matching tasks with goals for student learning. Take for example the case of Mr. Johnson (Silver & Smith, 1996). Mr. Johnson wanted his students to learn to work collaboratively, to discuss alternative approaches to solving tasks, and to justify their solutions. However, the tasks he tended to use (e.g., expressing ratios such as $15/25$ in lowest terms) provided little, if any, opportunity for collaboration, exploration of multiple solution strategies, or meaningful justification. Not surprisingly, class discussions were not very rich or enlightening. The discourse focused on correct answers and describing procedures, doing little to further students' ability to think or reason about important ideas associated with ratio and proportion.

Mr. Johnson's experience (and that of many teachers with whom we have worked) makes clear the need to start with a cognitively challenging task that has the potential to engage students in complex forms of thinking if the goal is to increase students' ability to think, reason, and solve problems. Although starting with such a task does not guarantee student engagement at a high level, it appears to be a necessary condition since low-level tasks virtually never result in high-level engagement (Stein, Grover, & Henningsen, 1996).

This is not to suggest that all tasks used by a teacher should engage students in cognitively demanding activity, since there may be some occasions on which

a teacher might have other goals for a particular lesson, goals that would be better served by a different kind of task. For example, if the goal is to increase students' fluency in retrieving basic facts, definitions, and rules, then tasks that focus on memorization may be appropriate. If the goal is to increase students' speed and accuracy in solving routine problems, then tasks that focus on *procedures without connections* may be appropriate. Use of these types of tasks may improve student performance on tests that consist of low-level items and may lead to greater efficiency of time and effort in solving routine aspects of problems that are embedded in more complex tasks. However, focusing exclusively on tasks of these types can lead to a limited understanding of what mathematics is and how one does it. In addition, an overreliance on these types of tasks could lead to the inability to apply rules and procedures more generally, that is, to similar but not identical situations, or to recognize whether a particular rule or procedure is appropriate across a variety of situations (NCTM, 1989). Hence, students also need opportunities on a regular basis to engage with tasks that lead to deeper, more generative understandings regarding the nature of mathematical processes, concepts, and relationships.

DIFFERENTIATING LEVELS OF COGNITIVE DEMAND

The Task Analysis Guide (shown in Figure 1.2) consists of a listing of the characteristics² of tasks at each of the levels of cognitive demand described earlier in the chapter: memorization, *procedures without connection*, *procedures with connections*, and *doing mathematics*. When applied to a mathematical task (in print form), this guide can serve as a judgment template (a kind of scoring rubric) that permits a "rating" of the task based on the kind of thinking it demands of students.

For example, the guide would be helpful in deciding that the Fencing Task (shown in Figure 1.2 in the Introduction) was an example of *doing mathematics* since the characteristics of this level most clearly describe the kind of thinking required to successfully complete the task. Specifically, no pathway is suggested by the task (i.e., there is no overarching procedure or rule that can simply be applied for solving the entire problem and the sequence of necessary steps is unspecified) and it requires students to explore pens of different dimensions and ultimately to make a generalization regarding the pen that will have maximum area for a fixed amount of fencing.

When determining the level of cognitive demand provided by a mathematical task, it is important not to become distracted by superficial features of the task and to keep in mind the students for whom the task is intended. Both of these considerations are discussed below.

THE TASK ANALYSIS GUIDE

Lower-Level Demands

Memorization Tasks

- involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
 - cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
 - are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
 - have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.
- Procedures Without Connections Tasks
- are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
 - require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
 - have no connection to the concepts or meaning that underlie the procedure being used.
 - are focused on producing correct answers rather than developing mathematical understanding.
 - require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands

Procedures With Connections Tasks

- focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
 - suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
 - usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
 - require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
- Doing Mathematics Tasks
- require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
 - require students to explore and understand the nature of mathematical concepts, processes, or relationships.
 - demand self-monitoring or self-regulation of one's own cognitive processes.
 - require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
 - require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
 - require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

FIGURE 1.2. The characteristics of mathematical tasks at each of the four levels of cognitive demand (Stein & Smith, 1998). (Reprinted with permission from *Mathematics Teaching in the Middle School*, copyright 1998 by the National Council of Teachers of Mathematics. All rights reserved.)

Going Beyond Superficial Features

Determining the level of cognitive demand of a task can be tricky at times, since superficial features of tasks can be misleading. Low-level tasks, for example, can appear to be high-level when they have characteristics of reform-oriented instructional tasks (NCTM, 1991; Stein et al., 1996) such as requiring the use of manipulatives; using "real-world" contexts; involving multiple steps, actions, or judgments; and/or making use of diagrams. For example, some individuals have considered Martha's Carpeting Task (shown in Figure 1.1 in the Introduction) a high-level task because it is a word problem and it is set in a real-world context. Similarly, some have considered commonly used fraction tasks—which ask students to find the sum of two proper fractions with unlike denominators and then to show the answer using fraction strips—high-level because they use manipulatives. But we would classify these tasks as low-level because typically well-rehearsed procedures (for Martha's Carpeting, the formula for determining area and for the fraction task, the rule for adding fractions with unlike denominators) are strongly implied by the problems. In both cases, the tasks would be considered to be *procedures-without-connections* tasks since there is little ambiguity about what has to be done or how to do it, there is no connection to concepts or meaning required, and the focus is on producing the correct answer.

It is also possible for tasks to be designated low-level when in fact they should be considered high-level. For example, the Lemonade Task—in which students have to determine which of two recipes for lemonade is more "lemony": Recipe A, which has 2 cups of lemon concentrate and 3 cups of water, or Recipe B, which has 3 cups of lemon concentrate and 5 cups of water—has been considered by some an example of a *procedures-without-connections* task because it "looks like" a standard textbook problem that could be solved by applying a rule or because it lacks "reform features" (such as requiring an explanation or justification). However, we have described this task as *doing mathematics* since no pathway for solving the problem is suggested (either explicitly or implicitly). Specifically, the task requires students to compare two situations and to determine which recipe has the higher proportion of concentrate. To do so, students must make sense of the problem situation and maintain a close connection to the meaning of ratio and to the question being asked. So even though tasks might "look" high- or low-level, it is important to move beyond their surface features to consider the kind of thinking they require.

Considering the Students

Another consideration when deciding the level of challenge provided by a task is the students (their age, grade level, prior knowledge and experiences) and

the norms and expectations for work in their classroom. Consider, for example, a task in which students are asked to add five two-digit numbers and explain the process they used. For a fifth- or sixth-grade student who has access to a calculator and/or the addition algorithm, and for whom "explain the process" means "tell how you did it," the task could be considered routine. If, on the other hand, the task is given to a second grader who has just started work with two-digit numbers, who has base-10 blocks available, and/or for whom "explain the process" means you have to explain your thinking, the task may indeed be high-level. Therefore, when teachers select or design instructional tasks, all of these factors must be considered in order to determine the extent to which the task is likely to provide an appropriate level of challenge for their students.

GAINING EXPERIENCE IN ANALYZING COGNITIVE DEMANDS

One way we have found to help teachers learn to differentiate levels of cognitive demand is through the use of a task-sorting activity. The long-term goal of this activity is to raise teachers' awareness of how mathematical tasks differ with respect to their levels of cognitive demand, thereby allowing them to better match tasks to goals for student learning. A task-sorting activity can also enhance teachers' ability to thoughtfully analyze the cases (which appear in Part II of this book), and ultimately, to become more analytic and reflective about the role of tasks in instruction.

The eight tasks shown in Figure 1.3 represent a subset of tasks we have used for this purpose. These tasks cover all four categories of cognitive demand and they vary with respect to a range of superficial features across these categories. For example, both tasks A and D require an explanation or description yet Task A is considered high-level (*doing mathematics*) and Task D is considered low-level (*procedures without connections*). Alternatively, both Tasks A and C are considered *doing mathematics*, yet they differ with respect to the use of manipulatives, a "real-world" context, and the use of a diagram.

Whether our tasks are chosen as the basis for a sorting activity, new tasks are created for this purpose, or some combination of tasks is used, it is important to vary tasks with respect to a range of features across categories of cognitive demand. Figure 1.4 provides a complete listing of the cognitive demands and features represented in the eight tasks shown in Figure 1.3. The analysis of tasks that vary in these ways will require going beyond superficial features to focus on the kind of thinking in which students must engage in order to complete the tasks.

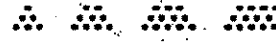

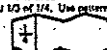

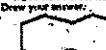
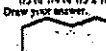


<p>TASK A</p> <p><i>Manipulatives/Tools:</i> Counters</p> <p>For homework Mark's teacher asked him to look at the pattern below and draw the figure that should come next.</p>  <p>Mark does not know how to find the next figure.</p> <p>A. Draw the next figure for Mark.</p> <p>B. Write a description for Mark telling him how you know which figure comes next.</p> <p><i>QUASAR Project - QUASAR Cognitive Assessment Instrument - Release Task</i></p>	<p>TASK E</p> <p><i>Manipulatives/Tools:</i> Pattern Blocks</p> <p>$\frac{1}{2}$ of $\frac{1}{2}$ means one of two equal parts of one whole.</p>  <p>Find $\frac{1}{2}$ of $\frac{1}{4}$. Use pattern blocks. Draw your answer.</p>  <p>Find $\frac{1}{4}$ of $\frac{1}{2}$. Use pattern blocks. Draw your answer.</p>  <p>$\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.</p>  <p>$\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.</p>  <p>$\frac{1}{4}$ of $\frac{1}{2}$ is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.</p>
<p>TASK B</p> <p><i>Manipulatives/Tools:</i> None</p> <p>Part A: After the first two games of the season, the best player on the girl's basketball team had made 12 out of 20 free throws. The best player on the boy's basketball team had made 14 out of 25 free throws. Which player had made the greater percent of free throws?</p> <p>Part B: The "better" player had to sit out the third game due to an injury. How many baskets (out of an additional 10 free throw "tries") would the other player need to make in order to take the lead in terms of greatest percentage of free throws?</p> <p><i>Adapted from Investigations Mathematics, Glencoe Macmillan/McGraw-Hill, New York, New York, 1994.</i></p>	<p>TASK F</p> <p><i>Manipulatives/Tools:</i> Square Pattern Tiles</p> <p>Using the side of a square pattern tile as a measure, find the perimeter (i.e., distance around) of each train in the pattern block figure shown below.</p>  <p>Train 1 Train 2 Train 3</p>
<p>TASK C</p> <p><i>Manipulatives/Tools:</i> Calculator</p> <p>Your school's science club has decided to do a special project on nature photography. They decided to take a little over 300 outdoor photos in a variety of natural settings and in all different types of weather. Eventually they want to organize some of the best photos into a display and enter the State nature photography contest. The club was thinking of buying a 35mm camera, but someone in the club suggested that it might be better to buy disposable cameras instead. The regular camera with autofocus and automatic light meter would cost about \$40.00 and film would cost \$3.98 for 24 exposures and \$5.95 for 36 exposures. The disposable cameras could be purchased in packs of three for \$20.00 with two of the three taking 24 pictures and the third one taking 27 pictures. Single disposables could be purchased for \$8.95. The club officers have to decide which would be the best option and they have to justify their decisions to the club advisor. Do you think they should purchase the regular camera or the disposable cameras? Write a justification that clearly explains your reasoning.</p>	<p>TASK G</p> <p><i>Manipulatives/Tools:</i> Grid Paper</p> <p>The pairs of numbers in a-d below represent the heights of stacks of cubes to be leveled off. On grid paper, sketch the front views of columns of cubes with these heights before and after they are leveled off. Write a statement under the sketches that explains how your method of leveling off is related to finding the average of the two numbers.</p>  <p>a) 14 and 8 b) 16 and 7 c) 7 and 12 d) 13 and 15</p> <p>By taking 2 blocks off the first stack and giving them to the second stack, I've made the two stacks the same. So the total # of cubes is now distributed into 2 columns of equal height. And that is what average means.</p> <p><i>[Taken from Visual Mathematics (Bennett & Foreman, 1989)]</i></p>
<p>TASK D</p> <p><i>Manipulatives/Tools:</i> None</p> <p>The cost of a sweater at J. C. Penney's was \$45.00. At the "Day and Night" sale it was marked 30% off of the original price. What was the price of the sweater during the sale? Explain the process you used to find the sale price.</p>	<p>TASK H</p> <p><i>Manipulatives/Tools:</i> None</p> <p>Give the fraction and percent for each decimal.</p> <p>.20 = _____ = _____</p> <p>.25 = _____ = _____</p> <p>.33 = _____ = _____</p> <p>.50 = _____ = _____</p> <p>.66 = _____ = _____</p> <p>.75 = _____ = _____</p>

FIGURE 1.3. Sample tasks that have been used in a sorting activity.

Developing a Shared Meaning

The benefits of a task-sorting activity, as described in the previous section, accrue not simply from completing the sort, but rather from a combination of small- and large-group discussions that provide the opportunity for conversation that moves back and forth between specific tasks and the characteristics of each category as illustrated in the Task Analysis Guide (Figure 1.2) and negotiating definitions for the categories. We have found that participants do not always agree with each other—or with us—on how tasks should be categorized, but that both agreement and disagreement can be productive.

For example, we have found that there is often complete (or near) consensus that Task E should be classified as *procedures with connections*. The discussion about the task often brings out the fact that the task focuses on *what it means* to take a fraction of a fraction (as opposed to using an algorithm such as “multiply the numerators and multiply the denominators”) and that it cannot be completed without cognitive effort (i.e., students have to think about what their actions mean as they work through the problem). For other tasks, such as Task A, there is often little agreement. Some consider Task A an example of *procedures without connections*, some as *procedures with connections*, and others as *doing mathematics*. The ensuing discussion often highlights the fact that there is no procedure or pathway stated or implied for Task A, yet the Task Analysis Guide has included the use of a procedure as a hallmark of tasks that were classified as *procedures without connections* and *procedures with connections*. A more focused look at the characteristics of *doing mathematics* can bring out the fact that tasks in this category require students to explore and understand the nature of relationships—a necessary step in extending and describing the pattern in Task A. The discussion generally concludes with teachers deciding it is a *doing-mathematics* task. By using the Task Analysis Guide as a template against which to judge this and other “little consensus” tasks, the group has a principled basis for the decisions they make.

It is easy to get side-tracked with discussions about how a specific group of students would solve a particular task or to become overly concerned about achieving complete consensus on every task. (This has happened to us more than once!) The goal is not to achieve complete agreement but rather to provide teachers with a shared language for discussing tasks and their characteristics and to raise the level of discussion among teachers toward a deeper analysis of the relationship between the tasks they select or create and the level of cognitive engagement that will be required of students. It is important to remind participants to consider the purpose of the task-sorting activity more generally—to begin to consider how and why tasks differ and how these differences can impact opportunities for student learning.

Task	Level of Cognitive Demand	Explanation of Categorization	Features
A	Doing mathematics	There is no pathway suggested by the task. The focus is on looking for the underlying mathematical structure.	<ul style="list-style-type: none"> •requires an explanation •uses manipulatives •involves multiple steps •uses a diagram •is symbolic/abstract •is “textbook-like”
B	Procedures with connections	The task focuses attention on the procedure for finding percents, but in a meaningful context.	<ul style="list-style-type: none"> •has “real-world” context •involves multiple steps •is “textbook-like”
C	Doing mathematics	There is no predictable pathway suggested by the task and it requires complex thinking.	<ul style="list-style-type: none"> •requires an explanation •has “real-world” context •involves multiple steps •uses a calculator •is “textbook-like”
D	Procedures without connections	The task requires the use of a well-established procedure for finding the sales price. There is no connection to meaning.	<ul style="list-style-type: none"> •requires an explanation •has “real-world” context •involves multiple steps •is “textbook-like”
E	Procedures with connections	The task provides a procedure for taking a fraction of a fraction but connects the procedure to meaning.	<ul style="list-style-type: none"> •uses manipulatives •involves multiple steps •uses a diagram •is symbolic/abstract
F	Procedures without connections	The task provides a procedure for finding the perimeter but requires no connection to meaning.	<ul style="list-style-type: none"> •uses manipulatives •uses a diagram •is symbolic/abstract
G	Procedures with connections	The task provides a procedure for finding the average that focuses on the underlying meaning of average.	<ul style="list-style-type: none"> •requires an explanation •involves multiple steps •uses a diagram •is symbolic/abstract
H	Memorization	The task requires the recall of previously learned information. No understanding is required.	<ul style="list-style-type: none"> •is “textbook-like”

FIGURE 1.4. Cognitive demands and features of the eight sample tasks.

Continuing to Differentiate Cognitive Demands

A task-sorting activity provides one way in which teachers can begin to differentiate cognitive demands of tasks. Two additional approaches, more closely connected to teachers' practice, can also be helpful in this regard.

One approach is to ask teachers to collect the tasks used during classroom instruction over a defined period of time (e.g., 3 weeks). Then the teachers can use the Task Analysis Guide to identify the cognitive demands of the tasks they collected and evaluate whether the collection provided sufficient opportunity for development of thinking, reasoning, and problem solving as well as basic skills.

Another suggestion is to have teachers use the Task Analysis Guide to evaluate the tasks in a unit or chapter of their textbook or instructional materials. This could lead to a discussion of the balance of instructional tasks provided by the materials and/or to having teachers rewrite the tasks that were identified as low-level so as to raise the level of cognitive demand of the task (i.e., change low-level tasks into high-level tasks).

MOVING BEYOND TASK SELECTION AND CREATION

This chapter has focused on analyzing the cognitive demands of tasks as they appear in curricular materials or as they are created by teachers. In the remainder of this book we will focus on the ways in which tasks that are set up at a high-level play out *during* classroom instruction. There are at least two reasons for restricting our focus to high-level tasks from this point forward. First, the widespread dissemination of the Standards documents (NCTM, 1989, 1991, 1995, 1998) has made teachers and staff developers keenly aware of the need to challenge student thinking and the recent publication of innovative curricula such as *Connected Mathematics* (Lappan, Fitzgerald, Friel, Fey, & Phillips, 1998) and *Mathematics in Context* (The Mathematics in Context Development Team, 1998) has provided teachers with access to a storehouse of challenging mathematical tasks. Hence high-level tasks are being used with increased frequency in our nation's classrooms. Second, in contrast to low-level tasks, which are almost always faithfully implemented, the enactment of high-level tasks is less predictable and often leads to unintended and unanticipated outcomes (Stein et al., 1996).

In the next chapter we will focus on understanding the complexities encountered when tasks leave the printed page and become entangled with the thoughts and actions of the teachers and students who give them life during classroom lessons. This is a critical part of the story for teachers who are

committed to ensuring that students are not only *exposed to*, but also *benefit from*, high-level tasks.

NOTES

1. The category *doing mathematics* includes many different types of tasks that have the shared characteristic of having no pathway for solving the task explicitly or implicitly suggested and therefore requiring nonalgorithmic thinking. This category includes tasks that are nonroutine in nature, are intended to explore a mathematical concept in depth, embody the complexities of real-life situations, or represent mathematical abstractions.

2. These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level thinking skills (1987), and from the examination and categorization of hundreds of tasks in QUASAR classrooms (Stein et al., 1996; Stein, Lane, & Silver, 1997).

